CHAPTER SEVEN

POLYGONS

Definition:

A polygon is a plane figure which is bounded by straight lines.

Polygons	
Number of sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

- A polygon has both interior as well as exterior angles.
- The interior angles of a polygon are those angles which lie within the polygon.
- The exterior angles of a polygon lie outside the polygon.
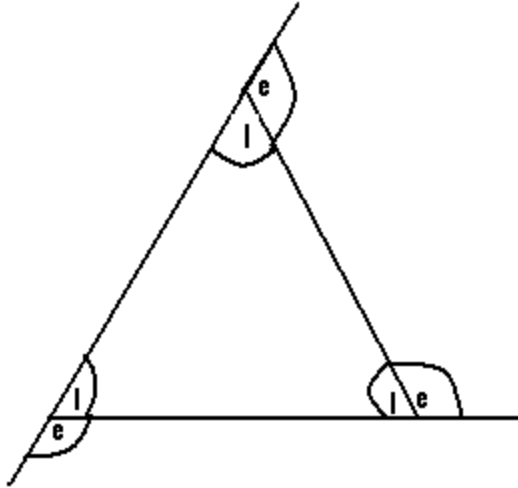
I = interior angle.

e = exterior angle.

N/B: For any polygon, the sum of the exterior angles = 360^0.

Q1. Calculate the value of each exterior angle of a regular decagon.

Soln.

Decagon has 10 sides and as such 10 exterior angles.

But the sum of the exterior angles of any polygon = 360^0.

$\Rightarrow 10$ exterior angles $= 360^0$.

$\therefore 1$ exterior angle $= \dfrac{1}{10} \times 360$

$\qquad = 36^0$.

$\Rightarrow$ each exterior angle of a decagon $= 36^0$.

Q2. Find the exterior angle of a regular pentagon.

Soln.

Pentagon has 5 sides, and as such 5 exterior angles. But the sum of the exterior angles of a polygon $= 360^0$

$\Rightarrow 5$ exterior angles $= 360$

$\Rightarrow 1$ exterior angle $= \dfrac{1}{5} \times 360$

$= 72^0$.

$\therefore$ Each exterior angle of the regular pentagon $= 72^0$. For any polygon, the sum of the exterior angle and the exterior angle at any of its vertices $= 180^0$.

<u>Determination of the interior angle of a regular polygon:</u>

- We must first determine the value of the exterior angle.

- Using the fact that at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow$ interior angle = 180^0 - exterior angle.

Q1. Calculate the interior angles of a regular decagon.

<u>Soln.</u>

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

= 36^0.

But at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow 36^0$ + interior angle = 180^0.

Interior angle = $180^0 - 36^0 = 144^0$.

The interior angle of the decagon = 144^0.

Q2. Find the value of each Interior angle of a triangle.

<u>Soln.</u>

A triangle has 3 sides and as such 3 exterior angles.

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360$

$= 120^0$.

But at any vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ Interior angle $+ 120^0 = 180^0$

$\therefore$ Interior angle $= 60^0$

<u>Determination of the sum or the total interior angles of a polygon:</u>

For any polygon, the sum of the interior angles = the number of sides of the polygon × the value of one interior angle.

Q1. Calculate the sum of the interior angles of a regular decagon.

<u>Soln.</u>

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles $= 360^0$

$\therefore$ 1 exterior angle $= \dfrac{1}{10} \times 360^0$

$$= 36^0.$$

But at any vertex, interior angle + exterior angle $= 180^0$

$\Rightarrow$ Interior angle $+ 36^0 = 180^0$

$\Rightarrow$ Interior angle $= 180 - 36$

$\Rightarrow$ Interior angle $= 144^0.$

But the sum of the interior angles of a decagon = interior angle × the number of sides.

$\therefore$ Sum of interior angles of the decagon $= 144^0 \times 10 = 1440^0.$

Q2. Find the sum of the interior angles of a regular octagon.

<u>Soln.</u>

Octagon has eight sides and as such eight exterior angles.

$\Rightarrow$ 8 exterior angles $= 360^0$

$\therefore$ 1 exterior angle $= \frac{1}{8} \times 360^0$

$\qquad\qquad = 45^0$.

But at any vertex, exterior angle + interior angle $= 180^0$

$\therefore 45^0$ + interior angle $= 180^0$

$\Rightarrow$ Interior angle $= 180 - 45 = 135^0$.

But the sum of interior angle = the number of sides of the polygon $\times$ interior angle $= 8 \times 135^0 = 1080^0$.

Q3. The interior angles of a regular triangle are marked $20^0 + 2x^0, 10^0 + 5x^0$ and $40^0 + 4x^0$. Find the actual values of each of these angles.

N/B: First calculate the sum of the interior angles of the triangle.

Soln.

Triangle has 3 exterior angles

$\Rightarrow$ 3 exterior angles $= 360^0$

$\therefore$ 1 exterior angle $= \frac{1}{3} \times 360^0$

$\qquad\qquad = 120^0$.

But at any vertex, exterior angle + interior angle $= 180^0$

$\Rightarrow 120^0$ + interior angle $= 180^0$

$\Rightarrow$ Interior angle $= 180^0 - 120^0 = 60^0$.

But the sum of the interior angles of the triangle = the number of sides $\times$ interior angle $= 3 \times 60 = 180^0$.

But the interior angles of the triangle are given as $20^0 + 2x^0, 10^0 + 5x^0$ and $40^0 + 4x^0$. The sum of these interior angles $= 20^0 + 2x^0 + 10^0 + 5x^0 + 40^0 + 4x^0$

$= 20^0 + 10^0 + 40^0 + 2x^0 + 5x^0 + 4x^0 = 70^0 + 11x$.

But the sum of the interior angles of the polygon or triangle $= 180^0$

$\Rightarrow 70 + 11x = 180^0$

$\Rightarrow 11x = 180^0 - 70 = 110^0$

$\Rightarrow x = \frac{110}{11} = 10^0.$

$\therefore$ The angle marked $20^0 + 2x = 20 + 2(10) = 20^0 + 20^0 = 40^0.$

The angle marked $10^0 + 5x^0 = 10^0 + 50(10) = 10 + 50^0 = 60^0.$

Lastly, the angle marked $40^0 + 4x^0 = 40 + 4(10) = 40 + 40 = 80^0.$

Q4. The angles of a pentagon are marked $x^0, (x^0 + 20^0), (x^0 + 25^0), 2x^0 \text{ and } (2x^0 + 5).$

   (a) Find the value of x.
   (b) Determine the value of each of those angles.
       Soln.

Pentagon has 5 exterior angles.

5 exterior angles $= 360^0$

$\therefore 1$ exterior angle $= \frac{1}{5} \times 360$

$\qquad\qquad = 72^0.$

But at any vertex, exterior angle + interior angle $= 180^0$

$\Rightarrow 72^0 + interior\ angle = 180^0$

$\Rightarrow$ interior angle $= 180 - 72 = 108^0.$

Sum of the interior angles of the pentagon = number of sides $\times$ interior angle

$= 5 \times 108 = 540^0.$

The given angles which are $x^0,\ x + 20^0,\ x + 25^0, 2x \text{ and } 2x + 5^0$ are the interior angles of the pentagon.

Sum of these interior angles $= x^0 + x + 20^0 + x + 25^0 + 2x + 2x + 5^0$

$= 7x + 50.$

Since the sum of the interior angles of the pentagon has been calculated to be equal to $540^0 \Rightarrow 7x + 50 = 540^0 \Rightarrow 7x = 540 - 50 \Rightarrow 7x = 490$

$\Rightarrow x = \dfrac{490}{7} = 70, \therefore \text{x} = 70^0.$

The value of the angle marked $x^0 = 70^0.$

The value of the one marked $\text{x} + 20^0 = 70 + 20 = 90^0.$

The angle marked $\text{x} + 25 = 70 + 25 = 95^0.$

The angle marked $2\text{x} = 2 \times 70 = 140^0.$

Lastly, the angle marked $2\text{x} + 5 = 2(70) + 5 = 140 + 5 = 145^0$